Multiple Iterated Belief Revision without Independence

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A motivating example for multiple belief revision

Adder and multiplier example

Suppose an electric circuit contains an adder and a multiplier. The atomic propositions a and m denote respectively that the adder and the multiplier are working. Initially we have no information about this circuit; then we learn that the adder and the multiplier are working: $A = a \wedge m$. Thereafter, someone tells us that the adder is actually not working: $B = \neg a$. At this point, established postulates for iterated revision imply that we have to *forget* that the multiplier is working because $B \models \neg A$.

Obviously, this problem occurs because AGM-style revision approaches can handle just one formula $a \wedge m$ instead of a set $\{a, m\}$.

Overview of this talk 1/2

We present approaches to belief revision that share basic ideas with AGM theory [Alchourron, Gaerdenfors & Makinson 1985] but goes far beyond $\rightarrow$ multiple and iterated belief revision:

- AGM theory and beyond

The main idea will be to overcome the limitations of AGM theory right away by considering belief revision in richer epistemic frameworks:

- Belief revision in probabilistics

Conditionals encode essential information for belief revision, and preserving conditional relationships is a main ingredient for advanced belief revision:

- The principle of conditional preservation for iterated and multiple belief revision

Overview of this talk 2/2

The principle of conditional preservation allows to transfer generic revision strategies to various semantic frameworks:

- Belief revision for ranking functions: c-revisions

This revision method for ranking functions should be evaluated appropriately:

- (Novel) Postulates for multiple iterated belief revision

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- (Novel) Postulates for multiple iterated belief revision
- Conclusion

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The core ideas of AGM theory

The AGM postulates are recommendations for rational belief change of a belief set K by new information A (within propositional logic):

- The beliefs of the agent should be deductively closed, i.e., the agent should apply logical reasoning whenever possible.
- The change operation should be successfull. (This does not mean that the agent should believe everything!)
- In case of consistency, belief change should be performed via expansion, i.e., by just adding beliefs.
- The result of belief change should only depend upon the semantical content of the new information.
- No change should be made if the new information is already known. (Minimal change paradigm resp. informational parsimony)

200 years before ...

Considering the task of belief change is not new: About 200 years before AGM theory, Bayes came up with his famous rule in probabilistics:

$$P(B|A) = \frac{P(A \wedge B)}{P(A)}.$$

Actually, Bayesian conditioning fulfills the core ideas of AGM theory, but obviously, the contexts of the theories (changing a code of law for AGM vs. random experiments and chances – e.g., in gambling – for Bayes) seemed to be too diverse to realise a strong connection.

The general task of belief change

However, from a formal resp. epistemic point of view, the tasks are similar if not identical:

General task of belief change

Given some (prior) epistemic state Ψ and some new information I, change beliefs rationally by applying a change operator $*$ to obtain a (posterior) epistemic state Ψ':

$$\Psi * I = \Psi'$$

AGM	: $\Psi = K$	set of propositional beliefs
Bayes	: $\Psi = P$	probability distribution
both	: $I = A$	propositional belief

Limitations of AGM theory

- Narrow logical framework: Classical propositional logic, no room for uncertainty
  $\rightarrow$ Richer epistemic frameworks
- One-step revision: AGM belief revision does not consider changes of epistemic states nor revision strategies
  $\rightarrow$ Iterated revision
- New information: Only one proposition – what about sets of propositions, conditional statements, sets of conditionals?
  $\rightarrow$ Conditional and multiple belief revision

Iterated belief revision and conditionals

Iterating belief revision means handling tasks of the form

$$((\Psi * A) * B) * C$$

(Ψ epistemic state, A, B, C formulas).

Via the Ramsey test

$$\Psi \models (B|A) \quad \text{iff} \quad \Psi * A \models B,$$

iterated revision is also about the revision of revision strategies, encoded by conditionals.

In iterative belief revision, the AGM principle of minimal change is replaced or complemented by a principle of conditional preservation [Darwiche & Pearl, AIJ 1997] which can be phrased in terms of conditionals.

A need for more powerful approaches to belief revision

Usually, plausible rules (conditionals) are essential parts of a rational agent's beliefs:

Birds' scenario

Birds fly, birds have wings. Penguins are birds but do not fly.
Kiwis are birds, doves are birds.
Inferences: Do kiwis and doves fly? Do penguins have wings?

New information: Kiwis do not have wings.
Inferences: Do kiwis fly?

This talk will present logic-based structures and general guidelines for iterated(, conditional) and multiple belief change.

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Probabilistic belief revision – a simple case

A simple probabilistic belief revision problem

The agent believes that the probability of A is $P(A)$, and now, she learns that B holds true – how should she change her belief on A?

. . . and a simple solution:

$$\boxed{P^{rev}(A) = P(A|B).}$$

– conditional probabilistic reasoning makes (simple) probabilistic belief revision easy.

Probabilistic belief revision – more complex cases

More complex belief revision problems:

- What if B holds with probability x with $0 < x < 1$? Here we have

Jeffrey's rule

$$P^{rev}(A) = x \cdot P(A|B) + (1 - x) \cdot P(A|\neg B)$$

- The agent wants to adapt her epistemic state P to a new conditional belief $(B|A)[x]$ – what is $P * (B|A)[x]$?
- The agent wants to adapt her epistemic state P to a set of new conditional beliefs $\{(B_1|A_1)[x_1], \ldots, (B_1|A_1)[x_1]\}$ – what is $P * \{(B_1|A_1)[x_1], \ldots, (B_1|A_1)[x_1]\}$?

The principle of Minimum cross-Entropy

Use cross-entropy = information distance (= Kullback-Leibler-divergence)

$$R(Q, P) = \sum_{\omega \in \Omega} Q(\omega) \log \frac{Q(\omega)}{P(\omega)}$$

ME belief change

Given some prior distribution P and some new information $\mathcal{R}$, choose the unique distribution

$$P^* = \mathit{ME}(P, \mathcal{R})$$

that satisfies $\mathcal{R}$ and has minimal information distance to P.

$\mathcal{R}$ may contain probabilistic conditionals as well as probabilistic and logical propositions.

The principle of minimum cross-entropy generalizes the principle of maximum entropy.

ME reasoning is somewhat familiar . . .

Probabilistic *ME* belief change has excellent properties, and it has long been known for special cases:

- for the simple case of adopting a certain belief $A[1]$, it coincides with Bayesian conditioning;
- for the more difficult case of adopting $A[x]$, it coincides with Jeffrey's rule.

Indeed, preserving conditional relationships with respect to prior P and new information $\mathcal{R}$ is one of the principal guidelines of *ME* belief change.

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Probabilistic principle of conditional preservation 1/2

Characteristics of conditionals:

- Conditionals are three-valued:

$$(B|A)(\omega) = \begin{cases} 1 & \text{if} \quad \omega \models AB \quad (\omega \text{ satisfies } A \text{ and } B) \quad (\textit{verification}) \\ 0 & \text{if} \quad \omega \models A\overline{B} \quad (\omega \text{ satisfies } A \text{ and } not\, B) \quad (\textit{falsification}) \\ u & \text{if} \quad \omega \models \overline{A} \quad (\omega \text{ satisfies } not\, A) \end{cases}$$

- The semantics of conditionals is determined by fractions:

$$P(B|A) = \frac{1}{1 + \frac{P(A\overline{B})}{P(AB)}}$$

Both characteristics play a major role for the principle of conditional preservation.

Probabilistic principle of conditional preservation 2/2

ME belief change satisfies the

Probabilistic principle of conditional preservation

Let $\Omega = \{\omega_1, \ldots, \omega_m\}$ and $\Omega' = \{\omega'_1, \ldots, \omega'_m\}$ be two sets of possible worlds[a] (not necessarily different).
If for each probabilistic conditional $(B_i|A_i)[x_i]$ in $\mathcal{R}$, Ω and Ω' behave the same, i.e., they show the same number of verifications resp. falsifications wrt $(B_i|A_i)$, then prior P and (*ME*-)posterior P^* are balanced by

$$\frac{P(\omega_1)\ldots P(\omega_m)}{P(\omega'_1)\ldots P(\omega'_m)} = \frac{P^*(\omega_1)\ldots P^*(\omega_m)}{P^*(\omega'_1)\ldots P^*(\omega'_m)}$$

[a] Possible world $\approx$ elementary event $\approx$ propositional interpretation

The principle of conditional preservation claims a balance between conserving conditional relationships in the prior epistemic state and establishing conditional relationships from the new information set.

A general principle of conditional preservation

The probabilistic principle of conditional preservation consists of two parts:

- A purely logical (or algebraic, resp.) precondition:

  For each probabilistic conditional $(B_i|A_i)[x_i]$ in $\mathcal{R}$, Ω and Ω' behave the same, i.e., they show the same number of verifications resp. falsifications

  – this has nothing to do with probabilities! (Cf. theory of conditional structures [GKI LNAI 2001])

- A semantical consequence:

$$\frac{P(\omega_1)\ldots P(\omega_m)}{P(\omega'_1)\ldots P(\omega'_m)} = \frac{P^*(\omega_1)\ldots P^*(\omega_m)}{P^*(\omega'_1)\ldots P^*(\omega'_m)}$$

We can use the algebraic precondition for any semantical framework that is rich enough to provide semantics for conditionals.

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AGM revision and total preorders

AGM revision needs to use total preorders on possible worlds as meta-information to make revision effective [Katsuno & Mendelzon 1991]:

$$\mathit{Mod}(\Psi * A) = \min(\mathit{Mod}(A), \leqslant_\Psi)$$

This holds even in the classical case when $\Psi = K$ is a belief set.

If natural (or ordinal) numbers are assigned to the strata of total preorders, we obtain ranking functions.

Ranking functions and conditionals

Ordinal conditional functions (OCF, ranking functions) [Spohn 1988]

$\kappa : \Omega \to \mathbb{N}(\cup\{\infty\})$ (Ω set of possible worlds, $\kappa^{-1}(0) \neq \varnothing$)

$\kappa(\omega_1) < \kappa(\omega_2)$ — ω_1 is more plausible than ω_2

$\kappa(\omega) = 0$ — ω is maximally plausible

$\kappa(A) = \min\{\kappa(\omega) \mid \omega \models A\}$, $\kappa(B|A) = \kappa(AB) - \kappa(A)$

Validating conditionals

$$\kappa \models (B|A) \text{ iff } \kappa(AB) < \kappa(A\overline{B}),$$

i.e., κ accepts a conditional $(B|A)$ iff its verification AB is more plausible than its falsification $A\overline{B}$.

Rankings can be understood as qualitative abstractions of probabilities, ranking theory is similar to possibility theory.

Advanced belief revision for ranking functions

Belief revision task for OCF

Given a prior OCF κ and some new information consisting of a set of conditionals $\Delta = \{(B_1|A_1), \ldots, (B_n|A_n)\}$, find a

$$\text{posterior OCF } \kappa^* = \kappa * \Delta$$

such that $\kappa^* \models \Delta$ and the revision complies with the core ideas of AGM.

This task involves

- iterated revision, since an epistemic state κ is changed;
- conditional revision, since the prior is revised by conditional information;
- multiple revision, since Δ can be a set of plausible propositions by setting $A \equiv (A|\top)$.

In the following, we will use ranking functions κ as typical representatives of epistemic states Ψ in the context of revision.

Multiple iterated revision for ranking functions

We will focus on this last case where we have propositions instead of conditionals:

Multiple revision task for OCF

Given a prior OCF κ and some new information consisting of a set of propositions $\mathcal{S} = \{A_1, \ldots, A_n\}$, find a

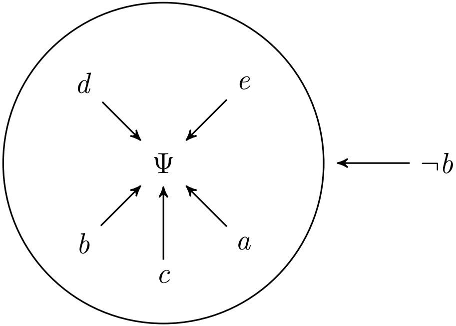
$$\text{posterior OCF } \kappa^* = \kappa * \mathcal{S}$$

such that $\kappa^* \models \mathcal{S}$ and the revision complies with the core ideas of AGM.

Note that $\kappa \models A$ iff $\kappa(\overline{A}) > 0$.

The problem of AGM with multiple iterated revision

Ψ epistemic state, e.g., $\Psi = \kappa$; a, b, c, d, e propositions:



AGM: $(\Psi * (a \wedge b \wedge c \wedge d \wedge e)) * \neg b \quad \not\models a, c, d, e$

More reasonably, $(\Psi * \{a, b, c, d, e\}) * \neg b \quad \models a, c, d, e$

Postulates for iterated revision

Postulates proposed by [Darwiche & Pearl 97]

(DP1) If $B \models A$, then $Bel((\kappa * A) * B) = Bel(\kappa * B)$.
(DP2) If $B \models \neg A$, then $Bel((\kappa * A) * B) = Bel(\kappa * B)$.
(DP3) If $A \in Bel(\kappa * B)$, then $A \in Bel((\kappa * A) * B)$.
(DP4) If $\neg A \notin Bel(\kappa * B)$, then $\neg A \notin Bel((\kappa * A) * B)$.

AGM revision amounts to using total preorders as meta-information for revision [Katsuno & Mendelzon 1991]. Hence iterated revision means revising total preorders $\preceq_\kappa$ (representing epistemic states κ):

($\mathrm{DP1}^{\mathrm{sem}}$) If $\omega_1, \omega_2 \models A$, then $\omega_1 \preceq_\kappa \omega_2$ iff [1] $\omega_1 \preceq_{\kappa * A} \omega_2$.
($\mathrm{DP2}^{\mathrm{sem}}$) If $\omega_1, \omega_2 \not\models A$, then $\omega_1 \preceq_\kappa \omega_2$ iff $\omega_1 \preceq_{\kappa * A} \omega_2$.
($\mathrm{DP3}^{\mathrm{sem}}$) If $\omega_1 \models A$ and $\omega_2 \not\models A$, then $\omega_1 \prec_\kappa \omega_2$ implies $\omega_1 \prec_{\kappa * A} \omega_2$.
($\mathrm{DP4}^{\mathrm{sem}}$) If $\omega_1 \models A$ and $\omega_2 \not\models A$, then $\omega_1 \preceq_\kappa \omega_2$ implies $\omega_1 \preceq_{\kappa * A} \omega_2$.

[1] $\omega_1 \preceq_\kappa \omega_2$ iff $\kappa(\omega_1) \leqslant \kappa(\omega_2)$

Do we need "Independence" for multiple iterated revision?

The Independence postulate of [Jin and Thielscher 07] has been deemed to be important for iterated revision and indispensable for multiple revision:

(Ind) If $\neg A \notin \mathit{Bel}(\kappa * B)$, then $A \in \mathit{Bel}((\kappa * A) * B)$.

(Ind$^{\text{sem}}$) If $\omega_1 \models A$ and $\omega_2 \not\models A$, then $\omega_1 \preceq_\kappa \omega_2$ implies $\omega_1 \prec_{\kappa * A} \omega_2$.

Independence means that revising by A should make a difference in any case (however, irrespective of logical dependencies wrt κ ![2]).

For multiple revision, Independence means that each $A_i \in \mathcal{S}$ should make a difference.

[2] Consider the case that $A \in \mathit{Bel}(\kappa)$, then $\kappa * A \neq \kappa$.

Postulates for (independence in) multiple revision

Generalization of (Ind$^{\text{sem}}$), (DP3$^{\text{sem}}$) and (DP4$^{\text{sem}}$) for multiple revision by [Delgrande & Jin 2012]:

- (Ret) If $A \in \mathcal{S}_1$, and for all $\mathcal{S}_c \subseteq \mathcal{S}_1$ which are consistent with $\mathcal{S}_2 (\neq \varnothing)$ we have $\neg A \notin Bel(\kappa * (\mathcal{S}_c \cup \mathcal{S}_2))$, then $A \in Bel((\kappa * \mathcal{S}_1) * \mathcal{S}_2)$.
- (Ret^{sem}) If[3] $\mathcal{S}|\omega_2 \subset \mathcal{S}|\omega_1$, then $\omega_1 \preceq_\kappa \omega_2$ implies $\omega_1 \prec_{\kappa * \mathcal{S}} \omega_2$.
- (PC3^{sem}) If $\mathcal{S}|\omega_2 \subseteq \mathcal{S}|\omega_1$, then $\omega_1 \prec_\kappa \omega_2$ implies $\omega_1 \prec_{\kappa * \mathcal{S}} \omega_2$.
- (PC4^{sem}) If $\mathcal{S}|\omega_2 \subseteq \mathcal{S}|\omega_1$, then $\omega_1 \preceq_\kappa \omega_2$ implies $\omega_1 \preceq_{\kappa * \mathcal{S}} \omega_2$.

In fact, (Ret) (or (Ind), respectively) are not mandatory for multiple revision. More specifically, independence of information is not the same as processing information independently under revision.

[3] $\mathcal{S}|\omega = \{A \in \mathcal{S} \mid \omega \models A\}$

How can probabilistic belief change help?

Remember: *ME* belief change satisfies the

Probabilistic principle of conditional preservation

Let $\Omega = \{\omega_1, \ldots, \omega_m\}$ and $\Omega' = \{\omega'_1, \ldots, \omega'_m\}$ be two sets of possible worlds[a] (not necessarily different).
If for each probabilistic conditional $(B_i|A_i)[x_i]$ in $\mathcal{R}$, Ω and Ω' behave the same, i.e., they show the same number of verifications resp. falsifications, then prior P and (*ME*-)posterior P^* are balanced by

$$\frac{P(\omega_1) \ldots P(\omega_m)}{P(\omega'_1) \ldots P(\omega'_m)} = \frac{P^*(\omega_1) \ldots P^*(\omega_m)}{P^*(\omega'_1) \ldots P^*(\omega'_m)}$$

[a] Possible world $\approx$ elementary event $\approx$ propositional interpretation

A principle of conditional preservation for multiple revision

Applying the principle of conditional preservation to multiple revision ensures independent processing of information in $\mathcal{S} = \{A_1, \ldots, A_n\}$:

OCF principle of conditional preservation

Let $\Omega = \{\omega_1, \ldots, \omega_m\}$ and $\Omega' = \{\omega'_1, \ldots, \omega'_m\}$ be two sets of possible worlds (not necessarily different).
If for each proposition A_i in $\mathcal{S}$, Ω and Ω' behave the same, i.e., they show the same number of verifications resp. falsifications, then prior κ and posterior $\kappa^* = \kappa * \mathcal{S}$ are balanced by

$$\begin{aligned}(\kappa(\omega_1) + \ldots + \kappa(\omega_m)) - (\kappa(\omega'_1) + \ldots + \kappa(\omega'_m)) \\ = (\kappa^*(\omega_1) + \ldots + \kappa^*(\omega_m)) - (\kappa^*(\omega'_1) + \ldots + \kappa^*(\omega'_m))\end{aligned}$$

Ranks correspond to logarithms of probabilities, so product/division correspond to sum/difference.

C-revisions

...are revisions that satisfy the principle of conditional preservation.

It can be shown that revisions $\kappa^* = \kappa * \mathcal{S}$ of the following form are c-revisions:

$$\kappa^*(\omega) = \kappa_0 \kappa_0 - \kappa(\mathcal{S}) - \kappa(\mathcal{S}) + \kappa(\omega) + \sum_{\substack{i=1 \\ \omega \models \overline{A_i}}}^{n} \kappa_i^- \tag{1}$$

with (non-negative) natural numbers $\kappa_1^-, \ldots \kappa_n^-$ satisfying

$$\kappa_i^- > \min_{\omega \models A_i} \{\kappa(\omega) + \sum_{\substack{j \neq i \\ \omega \models \overline{A_j}}} \kappa_j^-\} \min_{\omega \models A_i} \{\kappa(\omega) + \sum_{\substack{j \neq i \\ \omega \models \overline{A_j}}} \kappa_j^-\} \kappa(\mathcal{S}) \kappa(\mathcal{S}) - \min_{\omega \models \overline{A_i}} \{\kappa(\omega) \tag{2}$$

where κ_0 is a normalizing constant to ensure that κ^* is an OCF, and (2) ensures that $\kappa^* \models \mathcal{S}$ (Success).

OCF C-revisions – Example

C-Revising κ by $\mathcal{S} = \{a, b\}$: and by $\neg b$ afterwards:

ω	$\kappa(\omega)$	$\kappa * \mathcal{S}(\omega)$	$=$	$\kappa_1(\omega)$	$\kappa_1 * (\neg b)(\omega)$
ab	4	$-4 + \kappa(\omega)$	$=$	0	1
$a\overline{b}$	1	$-4 + \kappa(\omega) + \kappa_2^-$	$=$	1	0
$\overline{a}b$	1	$-4 + \kappa(\omega) + \kappa_1^-$	$=$	1	2
$\overline{a}\overline{b}$	0	$-4 + \kappa(\omega) + \kappa_1^- + \kappa_2^-$	$=$	4	3

$\kappa \models \overline{a}\overline{b}$, $\kappa(ab) = 4$;

$\kappa_1 = \kappa * \{a, b\} \models ab$ with $\kappa_1^- = \kappa_2^- = 4$;

$\kappa_1 * (\neg b) \models a\overline{b}$.

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Evaluating c-revisions

It can be shown easily that OCF c-revision satisfy all of the established postulates except for those that are related to Independence; in particular, they satisfy the extended AGM and DP postulates.

It can also be shown that they satisfy the following postulates that are novel in this form:

(Novel) Postulates for multiple iterated revision

(Stability I) If $\Psi \models \mathcal{S}$, then $\Psi * \mathcal{S} = \Psi$.
One of the most basic AGM postulates which is, however, not satisfied by many (multiple) iterated revision operators having been proposed so far.

(PosCoherence) Let $\mathcal{S}, \mathcal{S}_1$ be consistent sets of sentences such that $\mathcal{S}_1 \subseteq \mathcal{S}$. Then $\Psi * \mathcal{S} = (\Psi * \mathcal{S}_1) * \mathcal{S}$.
One of the characteristic axioms of the principle of minimum cross-entropy.

(NegCoherence) Let $\mathcal{S}, \mathcal{S}_1, \mathcal{S}_2$ be sets of sentences such that $\mathcal{S} = \mathcal{S}_1 \cup \mathcal{S}_2$ and $\mathcal{S}_1 \cup \overline{\mathcal{S}_2}$ are consistent[4]. Then $\Psi * (\mathcal{S}_1 \cup \overline{\mathcal{S}_2}) = (\Psi * \mathcal{S}) * (\mathcal{S}_1 \cup \overline{\mathcal{S}_2})$.
Full epistemic generalization of the (S) postulate of [Delgrande & Jin 2012].

[4] $\overline{\mathcal{S}} = \{\overline{A} \mid A \in \mathcal{S}\}$

Recommendation for multiple iterated revision

The following postulates seem to be particularly recommendable for multiple iterated revision:

- Extended AGM ($\rightarrow$ aka total preorder underlies epistemic state)
- Success
- Stability I
- PosCoherence and NegCoherence
- (PC3) and (PC4) from [Delgrande & Jin 2012]

Independence is a kind of enforcement, i.e., multiple occurrences of information count.

For OCF, it can be shown that (together with some mild conventions) the principle of conditional preservation (PCP) implies PosCoherence, NegCoherence, (PC3), and (PC4).

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Conclusion

- We showed how studying revision in richer epistemic frameworks helps to find more powerful revision strategies for multiple iterated revision.
- The principle of conditional preservation (PCP) proved to be a major guideline for iterated revision and induce constructive approaches to multiple iterated revision.
- The Independence postulates are not necessary for multiple revision – we presented an OCF revision operator that handles multiple pieces of information adequately without satisfying (Ind), or (Ret) (a generalization to multiple conditional revision is straightforward).
- The machinery can be used for inductive reasoning as well (revising a uniform state by a knowledge base).
- Current and future work:
  - Developing network techniques for ranking functions
  - Completing the axiomatic view on multiple iterated belief revision